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ABSTRACT

A comprehensive list of concepts and skills for mathematics in kindergarten through grade 8 is provided. The 350 items have been logically organized and sequenced under five major topics: (1) number systems; (2) sets; (3) geometry; (4) measurement; and (5) other mathematical topics. Concepts that are related are cross-referenced by the assigned computer code. The document is a basic reference tool which can be used in developing and evaluating mathematics curricula. (MNS)

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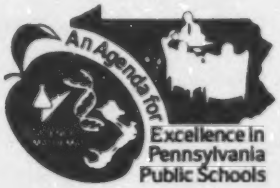
Mathematics Content List, Grades K-8 (Abridged)



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INTRODUCTION

Relation to State Mathematics/Science Plan:

The Mathematics Content List, Grades K-8 (MCL) is a comprehensive list of mathematics concepts/skills for kindergarten through grade eight. The MCL is an integral component of these initiatives outlined in The Pennsylvania Plan for the Improvement of Science and Mathematics Education:

- Diagnostic - Prescriptive Teacher Training in Elementary Mathematics, pages 8 and 11 - all components of the training materials including the teacher manual, instructor manual, student prescription catalog, diagnostic instruments, videotapes and manipulatives are correlated by the MCL codes.

- Chapter 5, Curriculum Regulations, page 9 - the MCL concepts/skills are the basis for selecting content for planned courses for grades kindergarten through eight.

- Mathematics Testing, Grades 3, 5, and 8 and Remedial Instruction, page 9 - the TELLS objectives and remedial instruction prescriptions are coded with the MCL concepts/skills for correlation to planned courses, Educational Quality Assessment (EQA), and the Pennsylvania Retrieval of Information for Managing Educational Systems (PRIMES).

- Instructional Media in Mathematics Education, page 12 - the MCL codes are the classification scheme for the PRIMES data base.

- Exemplary Mathematics Practices, page 13 - if applicable, these practices will be classified by the MCL codes.

Description:

The MCL contains 350 concepts and skills that have been logically organized and sequenced in standard outline form under five major topics.

- Number Systems -- includes sets of numbers; their structure, operations and properties; and the relationships among the sets.

- Mathematical Topics -- are topics included in a K-8 curriculum that cannot be classified discretely in the other four major topics because:

  .. they could be separate courses of study in later grades; e.g., history of mathematics, probability, logic;

.. they apply to the concepts/skills of all four topics; e.g., vocabulary, problem-solving, estimation;

.. they have traditionally been treated as separate topics; e.g., ratio and proportion, percent; or

.. they are techniques or devices used in mathematics that should be taught separately; e.g., calculators, flow charts.

. Sets -- are a fundamental idea to numerical and geometric concepts. Topic III describes members of sets and kinds of sets.

. Geometry -- includes sets of points; their properties and operations; and concepts of metric geometry.

. Measurement -- includes basic concepts about measurement; systems of measurement; and quantities measured.

There are two editions:

. Unabridged -- consists of four parts:

.. Content statement
.. Computer code of four digits
.. Definition, explanation or diagram
.. Example

. Abridged -- consists of two parts:

.. Content statement
.. Computer code of four digits

Example: Concept (abridged)

0410	a) Addition of whole numbers developed by using the union of disjoint sets or joining action (See 6850)
↑ Code	↖ Concept

Concepts that are related are cross referenced by the computer code. The computer codes are assigned in sequential order from 0010 - Definition of cardinal numbers, to 8700 - Other concepts of measurement and appropriate units. Gaps in the sequence permit insertion of new concepts. The numerical codes have no intrinsic meaning. The index references the computer codes of each concept/skill.

The MCL (unabridged) is the basic document for developing a mathematics curriculum using PRIMES. All PRIMES system components are correlated by the MCL codes and statements:

- Performance objectives
- Curriculum guide concept pages
- Computer-generated textbook indexes
- Computer-generated correlation reports
- Curriculum design records and forms

Since the MCL is arranged in a logical, not pedagogical, sequence users construct sequences which they judge appropriate for their programs. The MCL serves to explain the mathematics of the PRIMES system so that users will share a common understanding of the mathematics. The PRIMES system is administered by the Central Susquehanna Intermediate Unit.

History:

The first edition of concepts and skills was empirically developed in 1965 from scope and sequence charts of major mathematics textbook series, grades kindergarten through six.

In 1966 through 1968 the list was used to classify textbook lessons, page-by-page in eleven series, grades K-6 (77 books). It was also used by 19 school districts in Westmoreland and ARIN Intermediate Units to develop curriculum guides. Computer programs were designed for textbook and test indexes and objectives/textbooks/tests correlation reports.

In 1969 and 1971 major revisions were made in the list of concepts and skills and in the computer coding scheme.

In 1974 the MCL was extended to include topics for grades 7 and 8. The format was changed for easy user access; graphics and diagrams were improved and increased.

During 1974 through 1983 the MCL was extended to include development of:

- a special education resources guide,
- multi-media instructional kits in English and Spanish,
- the state assessment instrument for mathematics,
- a graduate level course in diagnostic prescriptive mathematics,
- a flexible grouping model for individualizing instruction,
- mathematics labs and clinics, and

. new computer software for indexes, correlation reports and curriculum guide concept pages.

Purposes:

The MCL is invaluable to all concerned with mathematics education -- administrators, supervisors, teachers, trainers of teachers, students, parents and school board members.

Some uses are specified.

Users	Uses
. Administrators	.. curriculum design and development, K-12
	.. planned courses for Pennsylvania Long Range Plan
	.. implementation of Pennsylvania Comprehensive Mathematics Plan (PCMP)
	.. correlation of curriculum, instruction, evaluation
	.. generalized curriculum development model
. Supervisors	.. selection of instruction and evaluation materials
	.. observation of instruction
	.. demonstration lessons
	.. correlation of instructional materials: textbooks, supplementals, computer software, ...
	.. comparison of student achievement
	.. plan staff development
. Teachers	.. plan for appropriate instruction
	.. grouping for individualized instruction
	.. management of student records, instructional materials

. College instructors	.. planning in methods courses
. Students	.. evaluation of achievement (self-evaluation)
. Parents	.. reporting student achievement
. School Board Members	.. establishing policy

MATHEMATICS CONTENT LIST: K-8 (abridged)

Topic I: Number Systems

A. Whole Numbers

1. Basic concepts

0010 . . . . . .a. Definition of cardinal numbers (number and numeral)

0020 . . . . . 1) Cardinal number zero
(See 6840)

0030 . . . . . 2) Number associated with the number line
(one-to-one correspondence)
(See 6810)

0040 . . . . . 3) Odd and even numbers
(See 0170)

b. Cardinal numbers: numeration and place value

0060 . . . . . 1) Cardinal numbers one through ten
(reading and/or writing numerals;
one-digit numerals)

0070 . . . . . 2) Cardinal numbers beyond ten (two-digit numerals)

0080 . . . . . 3) Three-digit numerals; place value

0090 . . . . . 4) Four or more-digit numerals; place value

0100 . . . . . 5) Expanded notation: whole numbers

0110 . . . . . 6) Rounding whole numbers
(See 6450)

0130 . . . . . c. Ordering whole numbers as greater than, less than, equal to or not equal to and between; and objects as fewer than or more than
(See 6810)

d. Number sequences

0150 . . . . . 1) Ordinal number sense and ordinal counting
(See 0160)

0160 . . . . . 2) Sequence of whole numbers increasing by one
(numbers before, after, between: 39, 40, 41, __, __, __; __59, __; 61, 62, __, __, 65, 66)
(See 0150, 0190)

0170 . . . . . 3) Skip counting: whole numbers (2's, 5's, 10's)
(See 0040, 0190, 0840, 1670)

0180 . . . . . 4) Backward counting: whole numbers (count down)

0190 . . . . . 5) Arithmetic progressions
(See 0160, 0170)

0200 . . . . . 6) Geometric progressions

0210 . . . . . 7) Triangular numbers

0220 . . . . . 8) Square numbers

0230 . . . . . 9) Factorial numbers

0240 . . . . . 10) Other whole number sequences

0260 . . . . . e Other names for whole numbers (general)

0280 . . . . . f. Historical development of whole numbers

g. Numeration systems

0300 . . . . . 1) Nondecimal place value systems; other whole number bases

0310 . . . . . 2) Roman numerals

0320 . . . . . 3) Other historical systems: whole numbers

0340 . . . . . h. General review; basic concepts of whole numbers

2. Operations

a. Addition

0400 . . . . . 1) Basic concepts: meanings and symbols of addition of whole numbers

0410 . . . . . a) Addition of whole numbers developed by using the union of disjoint sets or joining action
(See 6850)

0420 . . . . . b) Addition of whole numbers developed by using the number line

2) Computation

a) Two addends

0450 . . . . . (1) Elementary facts of addition: whole numbers; addition tables

0460 . . . . . (2) Addition of two multi-digit whole numbers without renaming

0470 . . . . . (3) Addition of two multi-digit whole numbers with renaming

b) More than two addends

0500 . . . . . (1) Addition of more than two single digit whole numbers

0510 . . . . . (2) Addition of more than two multi-digit whole numbers

0530 . . . . . c) Historical algorithms: addition of whole numbers

0540 . . . . . d) General review: addition of whole numbers

0550 . . . . . e) Addition of money

0580 . . . . . 3) Properties of addition of whole numbers (See 1500)

b. Subtraction

0600 . . . . . 1) Basic concepts: meaning and symbols of subtraction of whole numbers

0610 . . . . . a) Subtraction of whole numbers developed in relation to subsets or separating action (See 6830)

0620 . . . . . b) Subtraction of whole numbers developed from number line

0630 . . . . . c) Subtraction, the inverse of addition of whole numbers (relationship of addition and subtraction; missing addend)

2) Computation

0650 . . . . . a) Elementary facts of subtraction: whole numbers

0660 . . . . . b) Multi-digits in subtraction of whole numbers without renaming

0670 . . . . . c) Multi-digits in subtraction of whole numbers with renaming

0730 . . . . . d) Historical algorithms: subtraction of whole numbers

0740 . . . . . e) General review: subtraction of whole numbers

0750 . . . . . f) Subtraction of money

0780 . . . . . 3) Properties of subtraction of whole numbers
(See 1500)

c. Multiplication

0800 . . . . . 1) Basic concepts: meaning and symbols of multiplication of whole numbers

0810 . . . . . a) Multiplication of whole numbers developed from union of two or more equivalent sets

0820 . . . . . b) Multiplication of whole numbers developed from arrays

0830 . . . . . c) Multiplication of whole numbers developed from the number line

0840 . . . . . d) Multiplication of whole numbers developed as repeated addition
(See 0170)

0850 . . . . . e) Multiplication of whole numbers developed from Cartesian product sets
(See 6890)

2) Computation

a) Two factors

0870 . . . . . (1) Elementary facts of multiplication: whole numbers; multiplication tables

0880 . . . . . (2) Multi-digits in multiplication of whole numbers without renaming

0890 . . . . . (3) Multi-digits in multiplication of whole numbers with renaming; one-digit multipliers

0900 . . . . . (4) Multi-digits in multiplication of whole numbers with renaming; two- or more-digit multipliers

0910 . . . . . (a) Ten and its multiples as factors; whole numbers (10-90)

0920 . . . . . (b) Powers of ten and multiples of powers of ten as factors: whole numbers

0930 . . . . . b) Multiplication of whole numbers with more than two factors

0940 . . . . . c) A number expressed in exponential form as a factor: whole numbers (See 1300)

0950 . . . . . d) Historical algorithms: multiplication of whole numbers

0960 . . . . . e) General review: multiplication of whole numbers

0970 . . . . . f) Multiplication of money

0990 . . . . . 3) Properties of multiplication of whole numbers (See 1500)

d. Division

1000 . . . . . 1) Basic concepts: meaning and symbols of division of whole numbers

1010 . . . . . a) Division of whole numbers developed from partitioning into equivalent sets (See 6830)

1020 . . . . . b) Division of whole numbers developed as successive subtraction

1030 . . . . . c) Division of whole numbers developed from arrays

1040 . . . . . d) Division of whole numbers developed from the number line

1050 . . . . . e) Division, the inverse of multiplication of whole numbers (includes missing factors)

2) Computation

1070 . . . . . a) Elementary facts of division: whole numbers

1080 . . . . . b) Division of whole numbers: single digit divisors; no remainder

1090 . . . . . c) Division of whole numbers: single digit divisors; remainder

1100 . . . . . d) Division of whole numbers: multi-digit divisors (includes estimating quotients)

1110 . . . . . (1) Division by ten and multiples of ten (10-90)

1120 . . . . . (2) Division by powers of ten and multiples of powers of ten

1140 . . . . . e) Division with whole numbers expressed in exponential form

1150 . . . . . f) Historical algorithms: division of whole numbers

1160 . . . . . g) General review: division of whole numbers

1170 . . . . . h) Division of money

1180 . . . . . 3) Properties of division of whole numbers (See 1500)

e. Combined operations (addition, subtraction, multiplication, division of whole numbers)

1) Two sequential operations: whole numbers

1200 . . . . . a) Computation: two sequential operations with whole numbers

1210 . . . . . b) Money: two sequential operations

2) More than two sequential operations: whole numbers

1230 . . . . . a) Computation: more than two sequential operations with whole numbers

1240 . . . . . b) Money; more than two sequential operations

1260 . . . . . 3) Combined operations (mixed): whole numbers

f. Exponents

1300 . . . . . 1) Raising to powers and finding roots; exponential notation: whole numbers (See 0940)

1310 . . . . . 2) Scientific notation: whole numbers

1400 . . . . . g. Order of operations: whole numbers (See 6500)

1500 . . . . . 3. Properties of whole numbers; definitions and applications; emphasis on <u>property</u> not operation: closure, commutativity, associativity, distributivity, identity, trichotomy (See 0580, 0780, 0990, 1190)

4. Elementary number theory

a. Factors

1600 . . . . . 1) Factors: whole numbers

1610 . . . . . 2) Primes: whole numbers

1620 . . . . . 3) Composites: whole numbers

1630 . . . . . 4) Greatest common factor: whole numbers (Euclidean algorithm)

1640 . . . . . 5) Prime factorization: whole numbers
(See 1610)

1650 . . . . . 6) Rules for divisibility: whole numbers

b. Multiples

1670 . . . . . 1) Multiples: whole numbers
(See 0170)

1680 . . . . . 2) Least common multiple: whole numbers
(See 2260, 2470)

B. Nonnegative Rational Numbers (fractional numbers: common, decimal)

2000 . . . . . 1. Basic concepts of common fractions: definition, meaning, symbols

2010 . . . . . a. Fractions developed in terms of the operation, division

2020 . . . . . b. Fractions developed from arrays or subsets

2030 . . . . . c. Fractions developed as distances on the number line

2040 . . . . . d. Fractions developed from plane and solid regions

2050 . . . . . e. Fractions developed in other ways (clock, money, etc.)

2060 . . . . . f. Whole numbers as related to set of nonnegative rationals (fractional numbers)
(See 3010)

g. Equivalence

2070 . . . . . 1) Equivalent common fractions

2080 . . . . . 2) Equivalent mixed numerals

2090 . . . . . h. Counting fractions

2100 . . . . . i. Ordering fractions: greater than, less than; equal to; not equal to; between
(See 2950)

2110 . . . . . j. General review: basic concepts of common fractions

2. Operations with common fractions

a. Addition

1) Basic concepts

2200 . . . . . a) Addition of fractions developed from union of disjoint sets

2210 . . . . . b) Addition of fractions developed from the number line

2220 . . . . . c) Addition of fractions developed from plane or solid regions

2) Computation

2240 . . . . . a) Addition with common fractions, equal denominators (like fractions)

2250 . . . . . b) Addition with mixed numerals, equal denominators (like fractions)

2260 . . . . . c) Addition with common fractions, unequal denominators (unlike fractions)
(See 1680)

2270 . . . . . d) Addition with mixed numerals, unequal denominators (unlike fractions)
(See 1680)

2280 . . . . . e) General review: addition with fractions

2300 . . . . . 3) Properties: addition of common fractions
(See 2950)

b. Subtraction

1) Basic concepts

2400 . . . . . a) Subtraction of fractions developed in relation to subsets

2410 . . . . . b) Subtraction of fractions developed from the number line

2420 . . . . . c) Subtraction of fractions developed from plane or solid regions

2430 . . . . . d) Subtraction, the inverse of addition of fractions

2) Computation

2450 . . . . . a) Subtraction with common fractions, equal denominators (like fractions)

2460 . . . . . b) Subtraction with mixed numerals, equal denominators (like fractions)

2470 . . . . . c) Subtraction with common fractions, unequal denominators (unlike fractions)
(See 1680)

2480 . . . . . d) Subtraction with mixed numerals, unequal denominators (unlike fractions)
(See 1680)

2490 . . . . . e) General review: subtraction with fractions

2510 . . . . . 3) Properties: subtraction of common fractions
(See 2950)

c. Multiplication

1) Basic concepts

2600 . . . . . a) Multiplication of fractions developed from addition of two or more equal fractions

2610 . . . . . b) Multiplication of fractions developed from arrays or sets

2620 . . . . . c) Multiplication of fractions developed from the number line

2630 . . . . . d) Multiplication of fractions developed from plane and solid regions

2) Computation

2650 . . . . . a) Multiplication with common fractions

2660 . . . . . b) Multiplication with mixed numerals

2670 . . . . . c) General review: multiplication with common fractions and mixed numerals

2690 . . . . . 3) Properties: multiplication of common fractions; multiplicative inverse (reciprocal)
(See 2950)

d. Division

1) Basic concepts

2790 . . . . . a) Division of fractions developed from arrays or sets

2800 . . . . . b) Division of fractions developed from successive subtraction of two or more equal fractional numbers

2810 . . . . . c) Division of fractions developed from the number line

2820 . . . . . d) Division of fractions developed from plane and solid regions

2830 . . . . . e) Division, the inverse of multiplication with fractional numbers

2) Computation

2850 . . . . . a) Division with common fractions

2860 . . . . . b) Division with mixed numerals

2870 . . . . . c) General review: division with fractions

2890 . . . . . 3) Properties: division of common fractions (See 2950)

2910 . . . . . e. Sequential operations: common fractions.

2930 . . . . . f. Combined operations (mixed): common fractions

2950 . . . . . g. Properties of common fractions: closure, commutativity, associativity, distributivity, identity, trichotomy, unit fraction (law of $\frac{1}{b}$), equality, density (See 2300, 2510, 2690, 2890, 3800)

3. Basic concepts of decimals

3000 . . . . . a. Definition, meaning, symbols: decimals

3010 . . . . . 1) Decimals developed by extending place value numeration system for whole numbers (See 2060)

3020 . . . . . 2) Decimals developed in relation to common fractions (denominators powers of ten)

3040 . . . . . b. Place value: decimals

3050 . . . . . 1) Expanded notation: decimals

3060 . . . . . 2) Rounding decimals

c. Equivalent decimals

3080 . . . . . 1) Equivalent terminating decimals

3090 . . . . . 2) Equivalent repeating decimals

3100 . . . . . 3) Ordering decimals as greater than, less than or equal to

3120. . . . . d. General review: basic concepts of decimals

4. Operations with decimals

a. Addition

3200 . . . . . 1) Meaning, concept development: addition of decimals

3210 . . . . . 2) Computation: addition of decimals

b. Subtraction

3300 . . . . . 1) Meaning, concept development: subtraction of decimals

3310 . . . . . 2) Computation: subtraction of decimals

c. Multiplication

3400 . . . . . .1) Meaning, concept development: multiplication of decimals

3410 . . . . . .2) Computation: multiplication of decimals

d. Division

3500 . . . . . .1) Meaning, concept development: division of decimals

3510 . . . . . .2) Computation: division of decimals

3600 . . . . . e. Sequential operations: decimals

3700 . . . . . f. Combined operations (mixed): decimals

3800 . . . . . g. Properties of decimals: closure, commutativity, associativity, distributivity, identity, trichotomy, equality
(See 2950)

C. Integers

1. Basic concepts

4000 . . . . . a. Definition: set of integers

4010 . . . . . 1) Integers developed from the number line

4020 . . . . . 2) Integers developed from physical world situations

4040 . . . . . b. Ordering integers: greater than; less than; equal to or not equal to; between

4050 . . . . . c. Directed numbers: integers; absolute value

2. Operations

a. Addition

4100 . . . . . 1) Basic concepts: definition, meaning, symbols of addition of integers

4110 . . . . . a) Addition of integers developed from number line

4120 . . . . . b) Addition of integers developed from physical world situations

4150 . . . . . 2) Addition of integers

4180 . . . . . 3) Properties: addition of integers; additive inverse

b. Subtraction

4200 . . . . . 1) Basic concepts: definition, meaning, symbols of subtraction of integers

4210 . . . . . a) Subtraction of integers developed from number line

4220 . . . . . b) Subtraction of integers developed from physical world situations

4230 . . . . . c) Subtraction, the inverse of addition of integers

4250 . . . . . 2) Subtraction of integers

4280 . . . . . 3) Properties: subtraction of integers

c. Multiplication

4300 . . . . . 1) Basic concepts: definition, meaning, symbols of multiplication of integers

4310 . . . . . a) Multiplication of integers developed from number line

4320 . . . . . b) Multiplication of integers developed from physical world situations

4350 . . . . . 2) Multiplication of integers

4380 . . . . . 3) Properties: multiplication of integers

d. Division

4400 . . . . . 1) Basic concepts: definition, meaning, symbols of division of integers

4410 . . . . . a) Division of integers develcped from number line

4420 . . . . . b) Division of integers developed from physical world situations

4430 . . . . . c) Division, the inverse of multiplication of integers

4450 . . . . . 2) Division of integers

4480 . . . . . 3) Properties: division of integers

4500 . . . . . e. Sequential operations: integers

4550 . . . . . f. Combined operations (mixed): integers

4600 . . 3. Properties of integers

4700 . . 4. General review of integers

D. Rational numbers

5000 . . . . 1. Basic concepts: definition of a set of rationals

5010 . . . . . a. Rationals developed from number line

5020 . . . . . b. Absolute value: rationals

5030 . . . . . c. Ordering rationals

2. Computation

5100 . . . . . a. Addition of rationals

5110 . . . . . b. Subtraction of rationals

5120 . . . . . c. Multiplication of rationals

5130 . . . . . d. Division of rationals

5140 . . . . . e. Sequential operations: rationals

5150 . . . . . f. Combined operations (mixed): rationals

5180 . . . . 3. Properties of rationals

E. Real numbers

1. Basic concepts

5200 . . . . . a. Irrational numbers developed as non-repeating decimals

5210 . . . . . b. Rational approximations: real numbers

5220 . . . . . c. The Pythagorean theorem; construction of irrational numbers
(See 7570)

5230 . . . . . d. Special irrational numbers: π , e

2. Computation

5300 . . . . . a. Addition of real numbers

5310 . . . . . b. Subtraction of real numbers

5320 . . . . . c. Multiplication of real numbers

5330 . . . . . d. Division of real numbers

5340 . . . . . e. Powers and roots: real numbers

5350 . . . . . f. Sequential operations: real numbers

5360 . . . . . g. Combined operations (mixed): real numbers

5380 . . . . . 3. Properties of real numbers: density completeness (number line); other properties

F. Complex numbers

5400 . . . . 1. Development: complex numbers

5410 . . . . 2. Computation: complex numbers

5500 . . G. Relationships: Number Systems - natural numbers, whole numbers, rationals, integers, real numbers and complex numbers

Topic II: Mathematical Topics

A. Ratio and proportion

6000 . . . . 1. Ratio
(See 7440)

6010 . . . . 2. Direct and inverse variation: ratio and proportion

6020 . . . . 3. Proportion (including rate pairs)

B. Per cent

6100 . . . . 1. Meaning, vocabulary, equivalent percent notation

6110 . . . . . a. Percent developed through use of ratios

6120 . . . . . b. Percent developed through use of equations

6130 . . . . . c. Percent developed through use of the formula p = b x r (percentage equals base times rate)

6150 . . . . 2. Computation: per cent

C. Graphing
(See 6260)

6200 . . . . 1. Solution sets of equalities and inequalities on the number line

2. The coordinate plane

6210 . . . . . a. Ordered pairs on a coordinate plane

6220 . . . . . b. Solution sets of equalities and inequalities on a coordinate plane

6230 . . . . 3. Slope, intercept, etc.

D. Descriptive statistics

6250 . . . . 1. Frequency tables and charts

6260 . . . . 2. Graphs (bar, line, circle, dot, picture, etc.)

6270 . . . . 3. Measures of central tendency: average; mean; mode; median

6280 . . . . 4. Measures of variability: range, quartiles, percentiles, average deviation, standard deviation

6300 . . E. Permutations and combinations

F. Probability

6320 . . . . 1. Basic concepts: probability

6330 . . . . 2. Computation probability

6350 . . G. Other mathematical systems (finite or infinite): modular arithmetic (clock) groups, fields; rings; etc.

H. Logic

6370 . . . . 1. Basic concepts of logic including reasoning

6330 . . . . 2. Formal logic
(See 6860)

6400. . I. Relations and functions (includes What's My Rule and Function Machine) domain and range; properties

J. Estimation

6450 . . . . 1. Estimation: number of objects
(See 0110)

6460 . . . . 2. Estimation in measurement

6470 . . . . 3. Estimation: solutions in computation

6500 . . K. Mathematical sentences: writing equations and inequalities
(See 1400)

6520 . . L. Problem solving strategies: formulating and analyzing the problem; finding and interpreting the solution

6530 . . M. Consumer mathematics

6532 . . . . 1. Income, banking, credit

6534 . . . . 2. Taxes, insurance

6536 . . . . 3. Purchasing, budgeting

6550 . . N. Flow charts

6560 . . O. Calculators

6570 . . P. Computers

6580 . . Q. History of mathematics

R. Trigonometry

6600 . . . . 1. Definition of trigonometric ratios

6610 . . . . 2. Numerical trigonometry

S. Algebra

1. Equations or inequalities

a. Solving linear algebraic equations

6700 . . . . . 1) Linear equations: zero, the identity element for addition

6710 . . . . . 2) Linear equations: one, the identity element for multiplication

6720 . . . . . 3) Linear equations: the distributive property

6740 . . . . . b. Linear algebraic equations in one variable

6750 . . . . . c. Linear algebraic equations in two or more variables

6760 . . . . . d. Quadratic algebraic equations in one variable

6770 . . . . 2. Order of operations: algebraic expressions

6790 . . T. Mathematics vocabulary: meaning of specialized words and symbols

Topic III: Sets: Physical and Abstract

6800 . . A. Set members or elements; description of sets, attributes

B. Kinds of sets

6810 . . . . 1. Equivalent and non-equivalent sets (one-to-one correspondence)
(See 0030, 0130)

6820 . . . . 2. Equal (identical) and unequal sets

6830 . . . . 3. Subsets
(See 0610, 1010)

6840 . . . . 4. The empty set
(See 0020)

6850 . . . . 5. Disjoint; union of sets; intersection of sets
(See 0410)

6860 . . . 6. Venn diagrams
(See 6380)

6870 . . . . 7. Finite and infinite sets

6880 . . . . 8. Universal set, difference and complement

6890 . . . . 9. Cartesian product sets (cross products)
(See 0850)

6950 . . C. General review of sets

Topic IV: Geometry

A. Basic concepts of geometric figures and forms

7000 . . . . 1. Geometric figures in environment

7010 . . . . 2. Geometric designs, patterns, sequences

7020 . . . . 3. Spatial relations without measurement (size, position, betweenness)
(See 8210)

4. Topological concepts

7040 . . . . . a. Curves: simple, non-simple, closed, open (includes mazes)

7050 . . . . . b. Regions formed by simple closed curves

7060 . . . . . c. Simple closed surfaces

5. Sets of points in a plane

7080 . . . . . a. Point, line, line segment, ray, plane

7090 . . . . . b. Related lines: intersecting, parallel, skew, oblique,...

7100 . . . . . c. Angles; kinds of angles; regions formed by angles

7110 . . . . . d. Constructions with straight edge, compass, protractor

6. Plane figures

7130 . . . . . a. Polygons: general properties

7140 . . . . . 1) Triangles

7150 . . . . . 2) Quadrilaterals

7160 . . . . . a) Parallelograms

7170 . . . . . (1) Rectangles

7180 . . . . . (2) Squares

7190 . . . . . b) Non-parallelograms

7200 . . . . . 3) Other polygons

7220 . . . . . b. Circles

7. Three dimensional space

7240 . . . . . a. General properties of three dimensional figures and space

7250 . . . . . b. Pyramid

7260 . . . . . c. Prism

7270 . . . . . d. Cylinder

7280 . . . . . e. Cone

7290 . . . . . f. Sphere

7300 . . . . 8. Conic sections: the ellipse, the circle, the parabola, and the hyperbola

7330 . . . . 9. General review of geometric concepts

B. Metric geometry

1. Comparing sizes, shapes, distances

7400 . . . . . a. Congruency

7410 . . . . . b. Symmetry

7420 . . . . . c. Transformations

7430 . . . . . d. Similarity

7440 . . . . . 1) Similarity: scale drawing and reading scales (See 6000)

2. Measurement/calculation of geometric representations

7500 . . . . . a. Line segments with compass or straightedge (See 8230, 8310)

7510 . . . . . b. Angles with protractor, compass, other measuring device

7520 . . . . . c. Perimeter of polygons (See 8230, 8310)

7530 . . . . . d. Circumference of circles

7540 . . . . . e. Area of plane figures; definition, formula, calculation

7550 . . . . . f. Volume of solids; definition, formula, calculation

7560 . . . . . g. Surface area of solids; definition, formula, calculation

7570 . . . . . h. The Pythagorean theorem and the distance between two points (includes Pythagorean triples) (See 5220)

7580 . . . . . i. Combined measurements (mixed): geometric representations

7600 . . . . C. Operations with geometric figures: union and intersection

7700 . . . . D. Other geometric topics: separation of sets of points; locus of points; geometric notation; vectors; non-Euclidean geometries; optical illusions

Topic V: Measurement of Nongeometric Quantities

8000 . . . . A. Meaning of measurement (direct, indirect)

8100 . . . . B. Significant digits, precision and error in measurement

C. Systems of measurement

8200 . . . . . 1. Historical development of measurement: non-standard, customary, metric

8210 . . . . . 2. Non-standard measures: length, weight, volume (liquid) (See 7020)

3. Customary

8230 . . . . . a. Length in customary units

8240 . . . . . b. Volume, capacity in customary units (liquids)

8250 . . . . . c. Weight in customary units

8260 . . . . . d. Temperature - Fahrenheit

8270 . . . . . e. Dry measures in customary units

8280 . . . . . f. Quantity in customary units (dozen, gross, etc.)

4. Metric

8300 . . . . . a. Base units; prefixes of metric system

8310 . . . . . b. Length in metric units

8320 . . . . . c. Volume, capacity in metric units (liquids)

8330 . . . . . d. Mass in metric units (weight)

8340 . . . . . e. Temperature - Celsius (centigrade)

8380 . . . . . 5. General review: systems of measurement

D. Money

8400 . . . . . 1. Names and values of coins; combinations of coins

8420 . . . . . 2. Making change

8440 . . . . . 3. Other money concepts (foreign currency, conversions)

E. Time

8500 . . . . . 1. Clock

8520 . . . . . 2. Calendar

8540 . . . . . 3. Other time concepts (time zones, decades)

8560 . . . . . 4. Operations with time

8600 . . . . . F. Operations and conversions within a system cf measurement related to denominate numbers

8700 . . . . . G. Other concepts of measurement and appropriate units